The Divergence Theorem

1. Describe the boundary of each of the following solids. (Your description should be thorough enough that somebody reading it would have enough information to find the surface area of the boundary).

   (a) The solid $x^2 + 4y^2 + 9z^2 \leq 36$.

   (b) The solid $x^2 + y^2 \leq z \leq 9$.

   (c) The solid consisting of all points (x, y, z) inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 3$.

2. Let $\vec{F}(x, y, z) = \langle x^2, 2y, e^z \rangle$. Let $\mathcal{S}$ be the surface of the cube whose vertices are $(\pm 1, \pm 1, \pm 1)$, oriented with outward normals. Evaluate the flux integral $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$.

3. Let $\vec{F}(x, y, z) = \langle x^3, z^2, 3y^2 z \rangle$. Let $\mathcal{S}$ be the surface $z = x^2 + y^2$, $z \leq 4$ together with the surface $z = 8 - (x^2 + y^2)$, $z \geq 4$. Evaluate the flux integral $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$ if $\mathcal{S}$ is oriented with outward normals.

4. True or false: If $\vec{F}$ is a vector field whose divergence is 0 and $\mathcal{S}$ is any surface, then the Divergence Theorem implies that the flux integral $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$ is equal to 0.

5. Let $\mathcal{S}_1$ be the surface consisting of the top and the four sides (but not the bottom) of the cube whose vertices are $(\pm 1, \pm 1, \pm 1)$, oriented the same way as in #2. Let $\vec{F}(x, y, z) = \langle x^2, 2y, e^z \rangle$, as in #2. Evaluate the flux integral $\iint_{\mathcal{S}_1} \vec{F} \cdot d\vec{S}$. (Hint: Use #2.)

6. Let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = \left\langle z^3 \sin e^y, z^3 e^{x^2 \sin z}, y^2 + z \right\rangle$, and let $\mathcal{S}$ be the bottom half of the sphere $x^2 + y^2 + z^2 = 4$, oriented with normals pointing upward. Find $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$.